A 3D–PEEC Formulation Based on the Cell Method for Full-Wave Analyses with Conductive, Dielectric, and Magnetic Media

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A novel partial element equivalent circuit (PEEC) formulation for solving full-Maxwell's equations, with piecewise homogeneous and linear conductive, dielectric, and magnetic media, is presented. It is based on the Cell Method, which by using integral variables as problem unknowns, is naturally suited for developing circuit-like approaches such as PEEC. Volume meshing allows complex 3-D geometries, with electric and magnetic materials, to be discretized. EM couplings in the air domain are modelled by integral equations.

***Index Terms*—PEEC, Cell Method, integral equations, electromagnetic compatibility, interconnects.**

I. Introduction

INTEGRAL METHODS (IMs) are suitable for solving 2-D or 3-D high frequency electromagnetic (EM) problems including complex structures embedded in a large air domain [1][2]. The main drawback of IMs is that a dense linear system is obtained. This may be untreatable in the case of large-scale problems. The development of data compression techniques based, e.g., on H–matrices with adaptive cross approximation (ACA) is however boosting the research on integral methods [3][4]. Recently, the Cell Method (CM) has proven to be particularly suited to build IMs by formulating EM problems directly in terms of degrees of freedom (DoFs), enforcing thus element continuity [4]–[7].

Among different IMs the partial element equivalent circuit (PEEC) method has shown to be capable of handling large-scale EM problems, derived from the design and the prototyping of electronic devices such as filters, power converters, printed circuit boards (PCBs), and interconnects [8][9]. For instance, the compliance of a device with electromagnetic interference (EMI) standards requires an accurate modeling of all parasitic coupling effects. The original PEEC formulation consists in the discretization of the Electric Field Integral Equation (EFIE) by piecewise constant pulse basis functions in order to obtain an equivalent circuit of the electronic device. The advantages are manifold, i.e. an accurate modelling of EM interactions in the air domain and an easy integration with the external network which is particularly suited for design purposes. By formulating the field problem directly into an algebraic form, the CM is particularly suited for implementing PEEC formulations. So far 2-D PEEC CM-based formulations for the discretization of thin conductive structures have been proposed [10][11].

Recently, a face-element 3-D PEEC model, accounting for resistive, inductive, and capacitive effects with both conductors and dielectrics, has been presented [12]. Magnetic media, which are however required when modelling inductors on PCBs, have been considered only in [13]. The main idea of this work is to present a 3-D PEEC formulation, based on the CM, including conductive, dielectric, and magnetic materials. The purpose is to provide a fast EM simulator applicable for EMI problems and suitable for analyses ranging from extremely low to very high frequencies.

II. Integral Formulation

Let $\Omega = \bigcup_k \Omega_k$ be the *interior region*, i.e. the union of n bounded and connected subdomains $\Omega_k \subset \mathbb{R}^3$, $k = 1 \dots n$, which include conductive, dielectric, and magnetic materials. Let $\Omega^C = \mathbb{R}^3 \backslash \Omega$ be the *exterior region*, which is unbounded and includes field sources ($\Omega_0 \subset \Omega^C$). The interface between interior and exterior regions is thus $\Gamma = \partial\Omega = \Omega \cap \Omega_C$.

A. Magnetic and electric potentials

In order to obtain a closed-form solution of Maxwell's equations for linear and isotropic media equivalent dipole sources are introduced. Dielectric and magnetic media can be replaced in the free space by equivalent density distributions, i.e. the electric $\mathbf{P}$ and the magnetic $\mathbf{M}$ polarization densities.

EM field problems with piecewise homogeneous conductors of electric conductivity σ, dielectrics of electric permittivity ε, and magnetic materials of permeability μ are governed by:

$$\begin{aligned} &\mathbf{J} = \sigma\,\mathbf{E} && \text{in } \Omega_c \\ &\mathbf{D} = \varepsilon\,\mathbf{E} = \varepsilon_0\,\mathbf{E} + \mathbf{P} && \text{in } \Omega_d \\ &\mathbf{B} = \mu\,\mathbf{H} = \mu_0\,(\mathbf{H} + \mathbf{M}) && \text{in } \Omega_m, \end{aligned} \tag{1}$$

where $\mathbf{J}$ is the electric current density, $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field, $\mathbf{D}$ is the electric displacement, and $\mathbf{B}$ is the magnetic flux density. Space regions $\Omega_c, \Omega_d, \Omega_m$ (with empty intersections) indicate the conductive, dielectric, and magnetic domain, respectively. ε_0, μ_0 are ε, μ in free space. From (1) polarization densities are derived, i.e. $\mathbf{P} = \varepsilon_0\chi_d\mathbf{E}$, $\mathbf{M} = \chi_m\mathbf{H}$, where χ_d and χ_m are the electric and magnetic susceptibility.

By introducing equivalent sources, Maxwell's equations in $\mathbb{R}^3$ can be formulated in the frequency domain as [14]:

$$\begin{aligned} &\nabla \times \mathbf{E} = -i\omega\,\mathbf{B} \\ &\nabla \times (\mu_0^{-1}\mathbf{B}) = \mathbf{J_0} + \mathbf{J} + \mathbf{J}_m + \mathbf{J}_d + \partial_t(\varepsilon_0\mathbf{E}) \\ &\nabla \cdot (\varepsilon_0\mathbf{E}) = \rho + \rho_d \\ &\nabla \cdot \mathbf{B} = 0, \end{aligned} \tag{2}$$

where i is the imaginary unit, ω is the angular frequency, $\mathbf{J_0}$ is the source current density in Ω_0, and $\mathbf{J}_m = \nabla \times \mathbf{M}$, $\mathbf{J}_\mathrm{d} = \partial_t\mathbf{P}$ are the magnetic and the polarization current density in Ω_m and Ω_d. ρ and $\rho_d = -\nabla \cdot \mathbf{P}$ are the free and bound charge density, respectively. From the div-free condition, the magnetic flux density in (2) can be expressed as $\mathbf{B} = \nabla \times \mathbf{A}$, where $\mathbf{A}$ is the magnetic vector potential. The first equation in (2) thus becomes $\mathbf{E} = -i\omega\mathbf{A} - \nabla\varphi$, where φ is the electric scalar potential. By using Lorenz's gauge, Maxwell's equations, expressed in terms of potentials, become:

$$\Delta\, \mathbf{A} + k^2\, \mathbf{A} = -\mu_0\, (\mathbf{J_0} + \mathbf{J}_e + \mathbf{J}_m), \tag{3}$$
$$\Delta\, \varphi + k^2\, \varphi = -\varepsilon_0^{-1}\rho_e, \tag{4}$$

where $k = \sqrt{\omega^2\varepsilon_0\mu_0}$ is a constant parameter, $\mathbf{J}_e$ and ρ_e are the current and charge density in the electric domain $\Omega_e = \Omega_c \cup \Omega_d$ ($\mathbf{J}_e = \mathbf{J}$ in Ω_c or $\mathbf{J}_e = \mathbf{J}_d$ in Ω_d, $\rho_e = \rho$ in Ω_c or $\rho_e = \rho_d$ in Ω_d). By introducing the scalar free space 3-D Green function $g(x,y) = e^{-ik|x-y|}/(4\pi|x-y|)$, it can be proven that the integral solutions of (3) and (4) in Ω^C are:

$$\mathbf{A}(x) = \mathbf{A_0}(x) + \mu_0 \int_\Omega g(x,y)(\mathbf{J}_e(y) + \mathbf{J}_m(y))dy, \tag{5}$$
$$\varphi(x) = \varepsilon_0^{-1} \int_\Omega g(x,y)\, \rho_e(y)dy, \tag{6}$$

where the field $\mathbf{A_0}$ is generated by the source current density.

B. Cell Method discretization

The computational domain is discretized into a tetrahedral mesh (primal grid $\mathcal{G}_\Omega$, with N nodes and E edges). Dual grids $\tilde{\mathcal{G}}_\Omega$ and $\tilde{\mathcal{G}}_\Gamma$ are then defined on Ω and Γ by taking the barycentric subdivisions of the primal grids $\mathcal{G}_\Omega$ and $\mathcal{G}_\Gamma$, i.e. the restriction of $\mathcal{G}_\Omega$ to Γ. The *augmented dual grid* is built by joining volume and boundary grids as $\tilde{\mathcal{G}}_{\Omega\Gamma} = \tilde{\mathcal{G}}_\Omega \cup \tilde{\mathcal{G}}_\Gamma$ [15]. These grids are related to the following incidence matrices, describing the connectivity between elements: $\mathbf{D}_\Omega$ (volumes to faces on $\mathcal{G}_\Omega$), $\mathbf{C}_\Omega$ (faces to edges on $\mathcal{G}_\Omega$), $\widetilde{\mathbf{G}}_\Omega = \mathbf{D}_\Omega^\mathrm{T}$ (edges to nodes on $\tilde{\mathcal{G}}_\Omega$), $\mathbf{C}_\Gamma$ (faces to edges on $\mathcal{G}_\Gamma$), and $\widetilde{\mathbf{G}}_\Gamma = \mathbf{C}_\Gamma^\mathrm{T}$ (edges to nodes on $\tilde{\mathcal{G}}_\Gamma$). Operators $\widetilde{\mathbf{G}}_{\Omega_e}, \widetilde{\mathbf{G}}_{\Gamma_e}$ are restriction of $\widetilde{\mathbf{G}}_\Omega, \widetilde{\mathbf{G}}_\Gamma$ to domain Ω_e, and operator $\mathbf{C}_{\Omega_m}$ that one of $\mathbf{C}_\Omega$ to domain Ω_m.

The arrays of DoFs for the 3-D PEEC defined on the primal grid are currents on faces f_i, $\mathbf{j}_e = (J_i)_{\Omega_e}$, with $J_i = \int_{f_i} \mathbf{J} \cdot d\boldsymbol{S}$ in Ω_e and magnetizations on edges e_i, $\mathbf{m} = (m_i)_{\Omega_m}$, with $m_i = \int_{e_i} \mathbf{M} \cdot d\boldsymbol{l}$ in Ω_m. Those defined on the dual grid are magnetic vector potentials on edges $\tilde{e}_i$, $\tilde{\mathbf{a}}_e = (\tilde{a}_i)_{\Omega_e}$, with $\tilde{a}_i = \int_{\tilde{e}_i} \mathbf{A} \cdot d\boldsymbol{l}$ in Ω_e, magnetic fluxes on faces $\tilde{f}_i$, $\tilde{\mathbf{b}}_{\boldsymbol{m}} = (\tilde{b}_i)_{\Omega_m}$, with $\tilde{b}_i = \int_{\tilde{f}_i} \mathbf{B} \cdot d\boldsymbol{S}$ in Ω_m, and electric scalar potentials on nodes $\tilde{n}_i$, $\widetilde{\boldsymbol{\Phi}}_e = (\widetilde{\Phi}_i)_{\Omega_e}$, where $\widetilde{\Phi}_i = \varphi(x_{\tilde{n}_i})$ in Ω_e is evaluated by (6).

The coupling between Ω and Ω^C is obtained by imposing the electric and magnetic constitutive relationships in weak form:

$$\int_{\Omega_e} \mathrm{w}_i^f(x) \cdot (\hat{\sigma}^{-1}\, \mathbf{J}_e(x) - \mathbf{E}(x))dx = 0, \tag{7}$$
$$\int_{\Omega_m} \mathrm{w}_i^e(x) \cdot (\hat{\nu}^{-1}\, \mathbf{M}(x) - \mathbf{B}(x))dx = 0, \tag{8}$$

where w_i^f, w_i^e are face and edge vector basis functions. $\hat{\sigma}$ is the equivalent conductivity in Ω_e ($\hat{\sigma} = \sigma$ in Ω_c, $\hat{\sigma} = i\omega\varepsilon_0\chi_d$ in Ω_d) and $\hat{\nu} = \chi_m/\mu$ is the equivalent reluctivity in Ω_m.

By inserting $\mathbf{A}$ and φ provided by (5) and (6) into (7) and (8), the following matrix equations are obtained:

$$\mathbf{R}\, \mathbf{j}_e + i\omega\, \tilde{\mathbf{a}}_e + \widetilde{\mathbf{G}}_{\Omega_e}\widetilde{\boldsymbol{\Phi}}_e = -i\omega\tilde{\mathbf{a}}_{\mathbf{0},e} - \widetilde{\mathbf{G}}_{\Gamma_e}\widetilde{\boldsymbol{\Phi}}_{\Gamma_e}, \tag{9}$$
$$\mathbf{S}\, \mathbf{m} - \tilde{\mathbf{b}}_m = \tilde{\mathbf{b}}_{\mathbf{0},m}, \tag{10}$$

$\mathbf{R} = (R_{ij})_{\Omega_e}$, $\mathbf{S} = (S_{ij})_{\Omega_m}$ are consititutive matrices, with $R_{ij} = \int_{\Omega_e} \hat{\sigma}^{-1}\, \mathrm{w}_i^f(x) \cdot \mathrm{w}_j^f(x)\, dx$, $S_{ij} = \int_{\Omega_m} \hat{\nu}^{-1}\, \mathrm{w}_i^e(x) \cdot \mathrm{w}_j^e(x)\, dx$; $\tilde{\mathbf{a}}_e = \mathbf{L}_e\mathbf{j}_e + \mathbf{L}_m\mathbf{j}_m$, $\tilde{\mathbf{b}}_m = \mathbf{M}_e\mathbf{j}_e + \mathbf{M}_m\mathbf{j}_m$ are integral terms.

By imposing the electric and magnetic charge conservation as $\mathbf{D}_{\Omega_e}\mathbf{j}_e = -i\omega\, \mathbf{q}_{\boldsymbol{e}}$ and $\mathbf{j}_m = \mathbf{C}_{\Omega_m}\mathbf{m}$, and by discretizing (6) into $\widetilde{\boldsymbol{\Phi}}_e = \mathbf{P}_e\, \mathbf{q}_{\boldsymbol{e}}$, the final matrix system is assembled as:

$$\begin{bmatrix} \mathbf{R} + i\omega\mathbf{L} & i\omega\mathbf{L}_m\mathbf{C}_{\Omega_m} & \widetilde{\mathbf{G}}_{\Omega_e} \\ -\mathbf{M}_e & \mathbf{S} - \mathbf{M}_m\mathbf{C}_{\Omega_m} & \mathbb{O} \\ \mathbf{P}_e\mathbf{D}_{\Omega_e} & \mathbb{O} & i\omega\mathbb{I} \end{bmatrix} \begin{bmatrix} \mathbf{j}_e \\ \mathbf{m} \\ \widetilde{\boldsymbol{\Phi}}_e \end{bmatrix} = \begin{bmatrix} -i\omega\tilde{\mathbf{a}}_{\mathbf{0},\Omega_e} - \widetilde{\mathbf{G}}_{\Gamma_e}\widetilde{\boldsymbol{\Phi}}_{\Gamma_e} \\ \tilde{\mathbf{b}}_{\mathbf{0},\Omega_m} \\ \mathbb{O} \end{bmatrix} \tag{11}$$

The extended paper will describe the solution procedure for (11) and will provide examples of application to relevant cases.

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